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RCM POLICY: NEW METHOD of RESIDUAL LIFETIME PREDICTION according to ITEM'S ACTUAL OPERATING STATE

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ABSTRACT

Preventive Condition-Directed Maintenance (PCDM) is known to be the most efficient repair policy for equipment items (parts, units, assemblies, etc.) which are subject to stochastic deterioration with increasing failure rate and can be periodically inspected. The problem is how to predict an item's residual life (until the next failure) with the acceptable accuracy and consistency.

We consider a repairable and replaceable item or a single-unit system and suppose the future operating and environmental conditions to be similar to the ones when the learning statistics was obtained. Practically, this is a reasonable assumption.

An adequate deterioration model was developed and a new conditional distribution of remaining lifetime was created on its base. Expected remaining life, optimal time interval to next inspection, and other necessary parameters may be obtained using this probability function. Rather high prediction accuracy with 14% mean relative error was approved by computation tests. Monte-Carlo Simulation of the developed PCDM policy showed, that total repair and downtime cost per unit time for this strategy was 9 to 12% less than for the repair-at-failure and periodic repair policies.

1. INTRODUCTION: STATEMENT OF THE PROBLEM AND ITS PLACE IN THE RCM METHODOLOGY

A repairable or replaceable item (unit or assembly), which is intended to perform a definite function and has its own coverage, is under our consideration. As a rule, such an item operates as a part of a long-time using equipment or as a single-unit system. Item's failures are attributable to physical degradation and thus limit its useful life and determine increasing failure rate. An internal-combustion engine, an electrical motor, a tire, a gear are examples of actual objects.

Assume, item's current operating state can be identified by a critical performance parameter(s). The deterioration status, when such parameter exceeds predefined control limit is interpreted as a critical failure. We use a term "critical", as here we consider failures causing necessity of item's overhaul or replacement. We'll also refer to above mentioned parameter's control limit as to "critical" one. Assume, values of such critical parameter and current operating time can be easily obtained due to regular inspections.

The principal problem is to determine an optimal PCDM policy, i.e. the most efficient scheduling for item's overhaul (or replacement) according to its actual value of critical parameter and operating time. Total expected maintenance and downtime cost per unit time is usual criteria for optimization.

We'll show, that solving this problem for above described items is closely connected with the famous RCM methodology [1], [2], etc.

Assume, main failure modes have been already identified, critical performance parameters and their critical levels have been also determined. According to the RCM methodology [2, p.70] we should answer the following question sequence (we'll omit some questions, that are not relevant):

Q.1 "Can function degradation or loss resulting from this failure mode be deferred by servicing?"
- NO, as we speak about wear-out and similar types of physical deterioration.

Q.2 "Is function degradation evident to operator during routine operation?"
- YES for relatively simple items (e.g. tire's tread depth reducing can be easily identified by a driver). In this case condition monitoring and assessing necessity of the on-condition repair is recommended [2]. A new method of ascertaining efficiency of immediate preventive repair versus operating until next inspection is one of the main points of this paper.
- NO in other cases, when signs of an impending failure are not so evident to an operator, or reasons for observed function degradation cannot be determined without inspection.

Q.3 "Is function failure evident to operator during routine operation?"
- YES, as here we analyze critical failures.

Q.5 "Does failure degrade safety?"
- an answer depends on particular item's type and does not influence further analysis.

Q.6 "Does failure reduce availability below an acceptable level?"
- YES, as here we analyze critical failures.

Q.8 "Does failure rate increase substantially with operating or calendar time?"
- YES, as in this paper we analyze items with increasing failure rate.

Q.9 "Can the remaining safe, useful life be assessed?"
- YES, as prediction of the remaining useful life is the principal point of this paper, and we consider items exactly of such kind. In this case RCM instructs [2, pp.74-75] to perform scheduled inspection for condition assessment to determine residual useful life before the item should be repaired or replaced. This paper is intended to suggest a method for prediction of the optimal inspection interval and remaining useful life-time. The basic point for this method is to construct conditional failure probability function. Condition refers here to definite (current) technical state of an item, which is identified by known values of critical performance parameter and corresponding operating time.

2. Brief Scope of Methods of Failure Probability Prediction for Various Degradations

Let us state a general problem of how to predict survival probability of an item, which deteriorates continuously (monotone) during its normal operating.

Statistical ageing process data over the large operating interval $[0;\ t_L]$ and observed realization (i.e. time-series data) of this process *A(t)* for a particular item over the smaller operating interval $[0;\ t_c]$ are assumed to be known. We should determine a distribution of the probability that this realization does not exceed the predefined critical limit A_{1lm}, i.e. probability of event:

$$a(t) < A_{1lm}. \qquad (1)$$

For many operating items the process *A(t)* is actually monotone-nondecreasing (or nonincreasing) random function over working interval from the initial state to the critical one. For examples, growth of a crack, decreasing the thickness of both components of a friction pair (e.g. shaft and socket, two pinions, piston and cylinder, etc.), accumulating the solid sediment in lubrication. Such monotone processes are often identified by nonrandom function with random arguments. A trend function can be constructed on the base of actual data available using the Least Square Method (LSM) or other methods of statistical analysis. The simplest and most convenient function is a linear one, so that degradation process is determined as follows:

$$A(t) = C + Bt, \qquad (2)$$

where *C, B* are random variables with joint probability density function *f(C,B)*.

If failure occurs only when the inequality (1) does not hold, then the reliability (or survival probability) function is

$$R(t) = \iint_{C+Bt<A_{1lm}} f(C,B)\,dC\,dB = \int_{-\infty}^{\infty} dC \int_{-\infty}^{(A_{1lm}-C)/t} f(C,B)\,dB \qquad (3)$$

If variables *C, B* are normally distributed with means M_c, M_b and standard deviations σ_c, σ_b, survival probability is also normal:

$$R(t) = \Phi\left[\frac{A_{1lm}-M_c-M_b t}{\sqrt{\sigma_c^2+\sigma_b^2 t^2}}\right], \qquad (4)$$

where Φ denotes the standardized and tabulated normal distribution.

The known Bernstein probability distribution can be easily derived from this reliability function:

$$F = 1-R(t) = \Phi\left[\frac{t-(A_{1lm}-M_c)/M_b}{\sqrt{\sigma_c^2+\sigma_b^2 t^2}/M_b}\right] \qquad (5)$$

If a process trend is not linear, in some cases it can be approximated by the following model:

$$A(t) = C + B\mu(t). \qquad (6)$$

If variables *C, B* are again normally distributed and independent, reliability function will be like the distribution (4), where the argument *t* should be substituted by the term $\mu(t)$.

Similarly, other reliability distributions can be derived, but such methods of reliability prediction use only operating time as actual source information and does not allow to take into account actual parameter values. A further presented model was developed in order to improve accuracy and consistency of reliability prediction due to utilizing actual parameter values, obtained during series of inspections.

3. Deriving Distribution of Conditional Failure Probability Depending on Actual Operating State

First we should specify mathematical model to describe adequately item's deterioration process. Only such model can provide a consistent and accurate prediction of failure probability.

A lot of actual wear-out data was collected in order to construct the required model. Degradation processes of about 15 item types with their critical performance parameters (gap between a piston and cylinder in a diesel engine, thickness of pinion's teeth, depth of tire's tread, gap in bearings, etc.) were under observation. Stochastic data set for each random process consists of some 10 to 16 time-series measured at sampling time instants with definite inspection interval. Preliminary investigation of scatter diagrams for this data (performed in accordance with [3, p.701-706]) indicated, that in general it is not worthwhile to fit the widely used linear function to the wear-out processes. Furthermore, any process of such type appears to have fairly high correlation between its successive cross-sections [4].

Finally, a stochastic model of monotone change of a critical performance parameter which provides the best fit to actual degradation data is as follows:

$$A(t) = V_k t^{\alpha} + Z(t), \qquad (7)$$

where α is a power index, which indicates inherent mechanism of physical degradation of each critical performance parameter and, hence, is its constant attribute defining the sharp of the parameter's trend;

V_k is a random index of parameter speed change, which is considered to be constant for any particular item over the observed operating interval;

Z(t) is stationary normal random process, identifying actual data deviations from the trend of each realization.

The statistical analysis of actual time-series data allowed to identify the main properties of the process *Z(t)*, whose expectation function is obviously equal to zero. It was approved that time dependence of its variance is stochastically insignificant, and, hence, a process's standard deviation may be treated as constant attribute σ_z for each particular critical parameter. An autocorrelation period τ_{cor} also appeared to be constant attribute and can be estimated using the same actual data. So, the autocorrelation of this process may be approximated with a triangular-shaped (i.e. linear) function:

$$\rho_z(\tau) = \begin{cases} 1-\tau/\tau_{cor}, & \text{while } \tau = t_2 - t_1 < \tau_{cor} \\ 0, & \text{elsewhere} \end{cases} \qquad (8)$$

The power index should be estimated utilizing the entire data sample on definite critical performance parameter with the help of standard LSM for the power function. If we know not less than 4 values of critical parameter for a particular item, the speed index also can be imated accurately enough according to LSM:

$$V_k = \sum_{i=1}^{k} A_i t_i^{\alpha} \Big/ \sum_{i=1}^{k} t_i^{2\alpha} \qquad (9)$$

Once the adequate model of the random 1otone degradation has been determined, we can 1rt deriving distribution of conditional .lure probability, which depends on actual jree of critical performance parameter. Let :tical (physical) parameter change from its itial value A_{nom} to critical limit A_{lim}, and $A(t)$ its current value (obtained by inspection). : convenience we'll transform this range to 3 standard interval [0; 1] using the following 1ation:

$$a(t) = \frac{A(t) - A_{nom}}{A_{lim} - A_{nom}} \qquad (10)$$

Let us now take into account the condition, at critical parameter value is equal to a_c at e operating time t_c, that is

$$a(t_c) = V_k t_c^{\alpha} + Z(t_c) = a_c. \qquad (11)$$

.ence we get the expression for the condition

$$Z(t_c) = a_c - V_k t_c^{\alpha}. \qquad (12)$$

Let x denote a residual lifetime. The)nditional probability of a failure, which may :cur before the instant t_c+x, may be written as)llows:

$$P[t_c+x<T] = P[a(t_c+x)>1 \mid Z(t_c) = a_c - V_k t_c^{\alpha}] = \qquad (13)$$
$$= P[Z(t_c+x) > 1 - V_k \cdot (t_c+x)^{\alpha} \mid Z(t_c) = a_c - V_k t_c^{\alpha}].$$

Now we can derive this conditional robability for the two correlated and normally istributed random variables $Z(t_c)$, $Z(t_c+x)$, sing trivial transformation rules for onditional and truncated bivariate normal istribution [5]. We have to use here the runcated normal distribution to provide onotonicity of function (1). Finally, we get :he following distribution of the conditional :ailure probability:

$$Q[x \mid a(t_c) = a_c] = \frac{\Phi\left[\dfrac{V_k (t_c+x)^{\alpha} + \rho_z(x) \cdot (a_c - V_k t_c^{\alpha}) - 1}{\sigma_z \sqrt{1-\rho_z^2(x)}}\right]}{\Phi\left[\dfrac{V_k (t_c+x)^{\alpha} - \rho_z(x) \cdot (a_c - V_k t_c^{\alpha})}{\sigma_z \sqrt{1-\rho_z^2(x)}}\right]} \qquad (14)$$

Examples of this distribution and corresponding density function for a few sets of source data are presented in Figures 1 and 2.

Since the conditional failure distribution has been found, we can calculate mean remaining lifetime as its first central moment, its variance and other remaining life characteristics.

The relevant software for IBM PC/AT has been developed to test and investigate the failure distribution (14). It was thoroughly checked against actual inspection data and reasonable prediction accuracy has been approved: mean relative error of prediction has not exceed 14%.

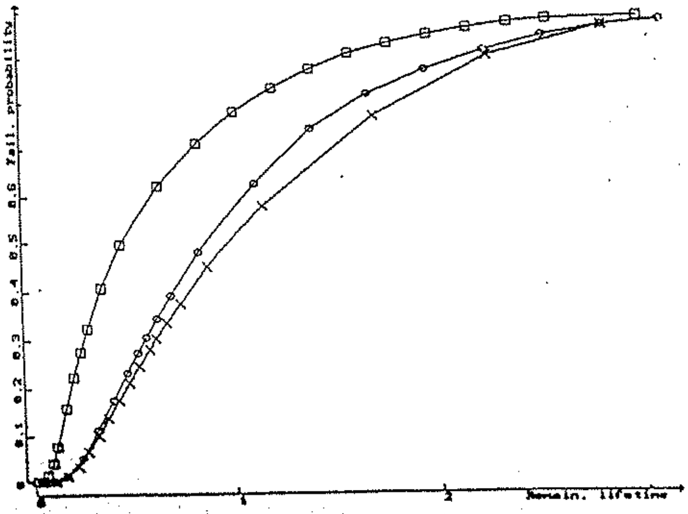

Figure 1. Conditional Failure Probability as a Function of Remaining Lifetime (Examples).

Source Input Data: $\alpha=1$; $\sigma_z=0.2$; $\tau_{cor}=1.5$; $V_k=0.22$
Current inspection result: (x) $t_c = 3.5$; $a_c = 0.75$
(o) $t_c = 5.0$; $a_c = 0.75$
(□) $t_c = 5.0$; $a_c = 0.85$

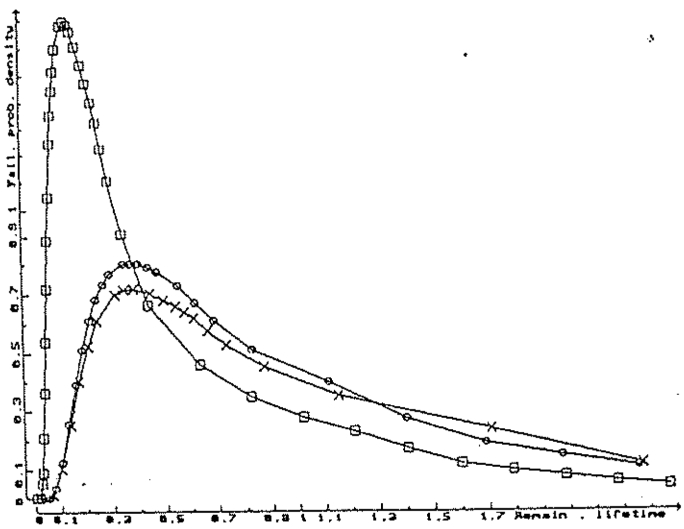

Figure 2. Conditional Failure Probability Density as a Function of Remaining Lifetime (Examples).

Source Input Data: $\alpha=1$; $\sigma_z=0.2$; $\tau_{cor}=1.5$; $V_k=0.22$
Current inspection result: (x) $t_c = 3.5$; $a_c = 0.75$
(o) $t_c = 5.0$; $a_c = 0.75$
(□) $t_c = 5.0$; $a_c = 0.85$

4. RELIABILITY-CENTERED PREVENTIVE MAINTENANCE MODELS

Once the distribution of the conditional failure probability of an item has been determined, we can create a few maintenance policies, differing by an item's kind and actually operating maintenance system.

If we consider a subsystem (or assembly) consisting of several items, much more PCDM policies can be constructed with the aim to decrease the total mean repair and downtime cost required at the maintenance level involved per unit operating time. The benefit might be obtained due to less quantity or/and duration of several maintenance activities (i.e. preparation, disassembly / reassembly, alignment, malfunction administrative time, etc.). For example, prior disassembly necessary to gain access to the required item, can be dramatically diminished. Furthermore, if we take into account all units of a system (equipment), we might significantly decrease corresponding system downtime.

Obviously, the described PCDM strategy can be classified as a variant of the well known Reliability Centered Maintenance (RCM) methodology [3], [4]. It is shown in [6], that RCM guidelines drive to the same problem of the condition assessment of an item's remaining lifetime (before its renewing).

To solve this problem, i.e. to construct a conditional failure probability function, it is necessary, first of all, to identify an adequate model of monotone degradation (aging). For many operating items the process $A(t)$ is actually monotone-nondecreasing (or nonincreasing) random function over working interval from the initial installation until an item can no longer perform its intended mission. For example, growth of a crack, decreasing the thickness of both components of a friction pair (e.g. two pinions, piston and cylinder, etc.), accumulating the solid sediment in lubrication.

Such monotone processes can be approximated by the mathematical model consisting of a nonrandom trend function $s(t)$ with random coefficients and a stochastic function which identifies actual random deviations. The simplest and most convenient function for trend approximation is a linear one, but statistical analysis of actual wear-out data showed, that in general the linear function does not adequately match the aging wear-out processes. Some mathematical functions (such as Exponential, Logarithmic, Fraction-linear, Power, etc.) are known to be used as the trend curves depending on actual degradation process. Several conditional failure probability distributions can be inferred on the base of certain deterioration model of this type for various trend function and properties of the random deviation process.

Based on collected wear-out sample data we built a stochastic model of the critical performance parameter degradation is a sum of a power trend function with a random index of parameter speed change V_k and a stationary random normal process, which identifies actual data deviations. After some mathematical transformations we obtained the following conditional failure probability distribution (its brief foundation is presented in the Appendix part of this paper):

(2)
$$Q[x \mid a(t_c)=a_c]=\Phi\left[\frac{V_k\cdot(t_c+x)^{\alpha}+\rho_{\varepsilon}(x)\cdot(a_c-V_k t_c^{\alpha})-1}{\sigma_{\varepsilon}\sqrt{1-\rho_{\varepsilon}^{2}(x)}}\right]/\Phi\left[\frac{V_k\cdot(t_c+x)^{\alpha}-\rho_{\varepsilon}(x)\cdot(a_c-V_k t_c^{\alpha})}{\sigma_{\varepsilon}\sqrt{1-\rho_{\varepsilon}^{2}(x)}}\right]$$

4. Software for Optimization of Current Preventive Maintenance Actions

In order to implement developed PCDM policies, it is necessary to perform regular prediction of an item's residual lifetime ccording to its current technical state and operating time. To fulfil this an appropriate software package for IBM PC ompatible computers was written (Microsoft C version 6.0). The program has user friendly interface with pull-down nested enus and self-explained input windows. Its main menu includes the following options:

Define data - input, update and delete all necessary constant and variable data.
Calculation - remaining lifetime prediction in "Test", "Research" and "Work" modes.
Results - textual reports and graphs.

Upon entering the program the main top-bar menu with these three options is displayed on the screen. At first, a user ould select the "Define data" option to be enabled to enter necessary data. Usually, input consists of constant and current ariable) information, and the program includes the two corresponding windows.

The first full-screen input window invites a user to enter such constant data, as Preventive and Failure repair cost, other ements of maintenance cost breakdown, nominal and critical performance parameter values, and so forth. The considered ftware package was created to optimize tires maintenance service, and respective input window is shown in Figure 3A.

On the second input window - "Inspection & Operation Condition Data" - a user will see results of previous inspections d will be prompted to enter current values of operating time and performance parameter, along with the current operation nditions. The last ones should be selected from the corresponding pop-up menus. Figure 3B depicts variant of this input ndow for tires (with the "Degree of Grip" pop-up menu as an example). It's worth mentioning, operating time for tires is asured as total run in kilometers, and tread depth is their critical performance parameter.

The "Calculation" menu is intended to fulfil prediction and includes the following options (Figure 4):

Test Run" is intended to verify software and validate current input. Actually, it does not save entered data in file, and erefore, permits to calculate result as many times as necessary, using one and the same previous inspection data.

Research Run" may be used for investigations, as it does not validate and restrict input at all. This option allows to perform nsitivity Analysis which is extremely useful to ascertain accuracy and consistency of the prediction method, as well as find out the reasons for possible errors.

Vork Run" should be used for regular operating. It performs a complete check of current inspection data. Specifically, the gram compares newly entered parameter value versus its previous one and the predefined control limits, as well as a rent operating time value should be greater than its previous one.

Each of these options provides three Prediction modes available (see Figure 4). A user may choose to calculate either